A Generalized Uniform Theory of Diffraction Method for Complex Illumination Sources Accounting for Multiply-Diffracted Rays

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***Abstract*— A new formalism is presented that generalizes the use of the Uniform Theory of Diffraction (UTD) to arbitrary illumination sources. The formalism returns the field in terms of a multipole expansion inside an extended observation region. When multiple UTD canonical objects are present, the multiply-diffracted rays can conveniently be superimposed. As an example, the scattering of a higher-order Hermite-Gaussian beam at a perfectly electrically conducting (PEC) slit is examined.**

I. Formalism

In this contribution, we consider two-dimensional transverse magnetic (TM) problems. For example, consider Fig. 1 in which diffraction occurs at a large scatterer, here an infinite PEC plate, which can be treated by UTD [1]. A phase center is attached to the arbitrary, distributed source, indicated by gray hatching in the figure, at a distance ρ'_c from the diffraction tip. The diffraction tip resides at an angle ϕ'_c from the phase center. The source is circumscribed by a circle $\mathcal{C}'$ with radius R'. Apart from the prime superscript, the same notation is used for the observation region.

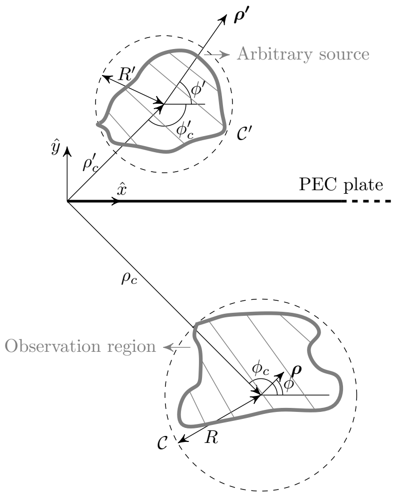


Fig. 1: Canonical problem geometry. The field from a spatially distributed source (hatched gray) is diffracted towards a spatially distributed observation region (hatched gray).

The diffracted field within the observation region is found by following a five-step procedure [2].

First, equivalent Huyghens' sources are placed on the boundary $\mathcal{C}'$.

Second, the source field E^{inc} at position $\boldsymbol{\rho}'$ from the source phase's center is expanded into multipoles, i.e.:

$$E^{inc}(\boldsymbol{\rho}') = \sum_{q'=-\infty}^{\infty} a_{q'} H^{(2)}_{q'}(k\rho') e^{jq'\phi'}, \tag{1}$$

where k is the wavenumber in the background medium, ϕ' is the angle that $\boldsymbol{\rho}'$ makes with the $\hat{x}$-axis and ρ' is the norm of the vector ($\rho' > R'$). As can be seen from (1), the coefficients $a_{q'}$ are proportional to the Fourier expansion coefficients of the sampled source field along $\mathcal{C}'$, such that they can easily be obtained once the source field is known. It has been shown previously that the coefficients $a_{q'}$ are also proportional to the coefficients of a Fourier expansion of the equivalent Huyghens' current [2].

Third, once the equivalent current is determined, UTD is applied for each source on the boundary $\mathcal{C}'$ and the diffracted field is determined by superposition.

Fourth, the diffracted field E^{diff} is expanded into multipoles within the observation region:

$$E^{diff}(\boldsymbol{\rho}) = \sum_{q=-\infty}^{\infty} b_q J_q(k\rho) e^{jq\phi}, \tag{2}$$

where $\boldsymbol{\rho}$ is a vector pointing from the observation's phase center towards an observation point and $\rho < R$.

In the fifth and last step, the equivalence of the diffracted field found by applying UTD and expansion (2) is imposed. This finally leads to a relationship between the coefficients b_q and $a_{q'}$ through coupling coefficients $t_{qq'}$, i.e.

$$b_q = \sum_{q=-\infty}^{\infty} t_{qq'} a_{q'}. \tag{3}$$

In practice, the infinite sums are truncated symmetrically. The truncations $q \in [-Q, Q]$ and $q' \in [-Q', Q']$ depend linearly on the radii R and R' respectively. The coupling coefficients are derived in [2]. Physically, they arise from a two-dimensional Fast Fourier Transform of the field, excited by sources on the boundary $\mathcal{C}'$, at observation points on boundary $\mathcal{C}$.

At transition regions, taking the Fast Fourier Transform of the discontinuous diffracted field can lead to inaccurate results. More accurate results are obtained by rewriting the *total* field in a form compatible with UTD and employing formulas (2)-(3) for the total field.

As demonstrated next, in the presence of multiple scatterers, this approach can still be used and the different diffraction contributions are superimposed during the computation of the coefficients $t_{qq'}$.

II. Results

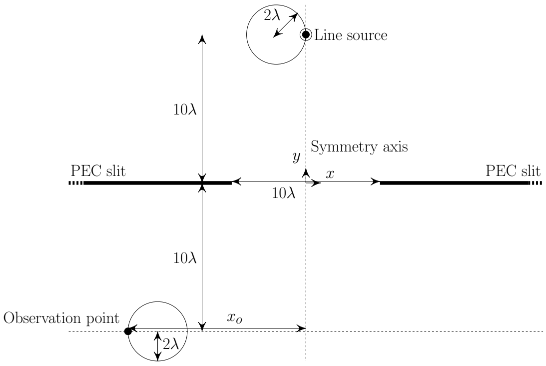


Fig. 2: PEC slit excited by a line source.

The accuracy of the method is assessed for the example shown in Fig. 2. A PEC slit of width 10λ, λ being the wavelength, is symmetrically illuminated by an electric line source and a phase center is chosen at a distance 2λ from the source. Different observation points are chosen on a line parallel to the slit at a distance of 10λ, at x-coordinates $x_o \in [-23\lambda, 22\lambda]$. A phase center is assigned to each observation point at a distance 2λ to its right. The slit is modelled as existing of two semi-infinite straight PEC plates, such that classical UTD can be employed. The interaction between the two tips of the slit is taken into account by considering double diffractions as a product of separate UTD contributions [3].

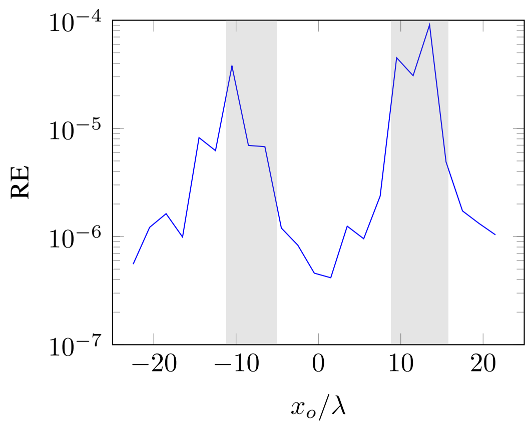


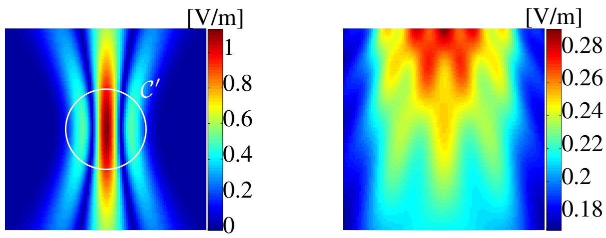
Fig. 3: Accuracy compared to classical UTD for the example of Fig. 2. Transition regions are indicated by a gray background.

The relative error (RE), defined as

$$\mathrm{RE} = \left| \frac{E^{tot} - E^{tot}_{UTD}}{E^{tot}_{UTD}} \right|, \tag{4}$$

is plotted in Fig. 3. E^{tot} is the total field computed by our method and E^{tot}_{UTD} is the reference UTD solution. The RE remains below 10^{-4} in this case, even in the transition regions.

Next, we treat the scattering of a 2^{nd} order Hermite-Gaussian beam at a PEC slit. Our method is able to deal with such complex illumination sources, as a set of sampled values of the source field on the boundary $\mathcal{C}'$ is already sufficient to determine the coefficients $a_{q'}$ in (1). The same slit as in Fig. 2 is used. The Hermite-Gaussian beam propagates along the symmetry axis of the slit. The beam has a waist λ at ten wavelengths above the slit. In order to determine the coefficients $a_{q'}$, the source field is sampled on a circle $\mathcal{C}'$ with radius 2λ. The total field behind the slit is computed in 10201 observation points within a square with $x \in [-5\lambda, 5\lambda]$ and $y \in [-15\lambda, -5\lambda]$. The computation took 47 s on a dual-core machine at 2.66 GHz with 8 GB of RAM.



(a) Incoming field.

(b) Total field behind slit.

Fig. 4: Field amplitude for diffraction of a Hermite-Gaussian beam at a PEC slit.

The amplitude of the Hermite-Gaussian beam near the source region is shown in Fig. 4a and the total field amplitude behind the slit is plotted in Fig. 4b.

III. Conclusions

In this contribution, we presented a novel method to treat diffraction by UTD for arbitrary illumination sources. The method can be used in the presence of multiple diffracting edges as multiply-diffracted rays are conveniently superimposed.

Acknowledgment

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References

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